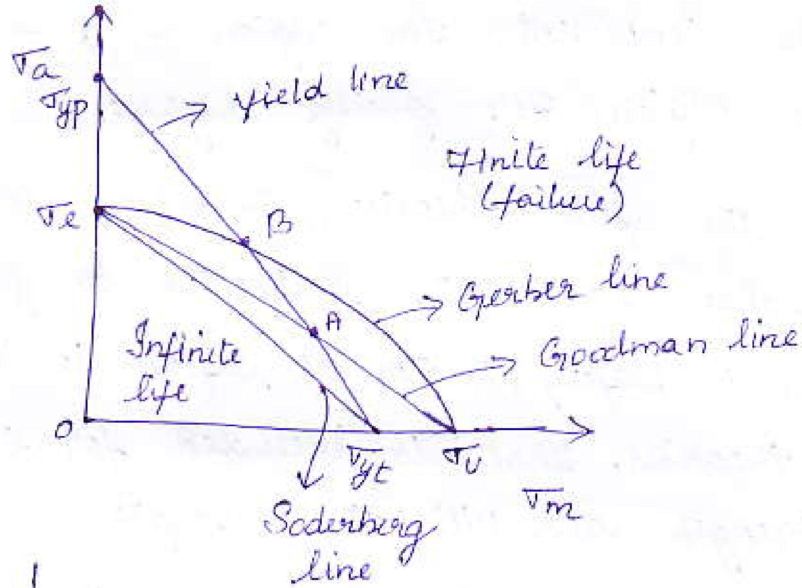
UNIT-I

Fatigue failure theories:

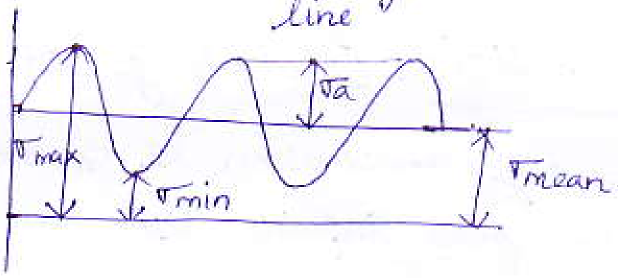
1. Goodman relation
2. Soderberg relation
3. Gerber relation

Relation between these theories



σ_a - Amplitude stress

σ_{yt} - Yield strength (σ_{yp})



Soderberg line:

It is the line b/w endurance limit (σ_e) and Yield point (σ_{yt}). The triangle $\underline{\sigma_e \, O \, \sigma_{yt}}$ is The safest limit. The object designed inside the Δ^{le} has infinite life.

Goodman line:

It is the line b/w endurance limit (σ_e) and ultimate strength (σ_u). The Triangle $\underline{\sigma_e \, O \, \sigma_u}$ is the infinite region

Modified Goodman:

It is the design b/w endurance limit (σ_e), intersection point (A) and σ_{yt}, and O. This region is infinite life region.

Gerber's criteria:

It is a parabolic curve drawn b/w σ_e & σ_u. The region $\boxed{\sigma_e O \sigma_u}$ is the infinite life region.

Gerber's yield criteria: [Modified Gerber];

This method consider yield point B, yield strength (σ_{yt}) for designing the component. The region σ_e & σ_u & 'O' is the infinite life region. ~~For giving highest~~

Among the five theories, Soderberg is the more safest design when components subjected to fatigue loads. For giving highest safety, Factor of Safety is included in all the above theories. That is included for endurance limit, yield strength and ultimate strength.

Derivation for Soderberg criteria: [Soderberg eqn]

In Soderberg theory any combination of σ_m and σ_a acting inside the triangular region gives infinite life.

Conditions are

If $\sigma_a = 0$, it is static load.

So when $\sigma_a = 0$, only static load is acting on the component. Now we can design either within σ_{yt} or σ_u.

When $\sigma_m = 0$, completely reversed fatigue load will act on the component. Now the design limit is based on σ_e.

The Soderberg eqn is derived using straight line

$y = mx + c$; Let $y = \sigma_a$; $x = \sigma_m$

m & c are constant, instead here we use 'a' & 'b'.

$\therefore$ Soderberg line equation is

$$\sigma_a = a\sigma_m + b \rightarrow (1)$$

To find the constants 'a' & 'b'.

The conditions are

when $\sigma_a = 0$; $\sigma_m = \sigma_{yt} \rightarrow (2a)$

when $\sigma_m = 0$; $\sigma_a = \sigma_e \rightarrow (2b)$

Substituting these conditions in eqn (1)

(2a) in (1) $\Rightarrow$ $0 = a\sigma_{yt} + b$

$$\boxed{b = -a\sigma_{yt}}$$

(2b) in (1) $\Rightarrow$ $\sigma_e = 0 + b$

$$\boxed{b = \sigma_e}$$

$\therefore$ $\sigma_e = -a\sigma_{yt} \Rightarrow \boxed{a = -\dfrac{\sigma_e}{\sigma_{yt}}}$

Now sub 'a' & b in eqn (1)

$$\sigma_a = -\frac{\sigma_e}{\sigma_{yt}} \cdot \sigma_m + \sigma_e$$

$$\sigma_a = \sigma_e\left[1 - \frac{\sigma_m}{\sigma_{yt}}\right]$$

$$\frac{\sigma_a}{\sigma_e} = 1 - \frac{\sigma_m}{\sigma_{yt}} \Rightarrow \boxed{\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{yt}} = 1} \rightarrow (2)$$

Equation (3) is the Soderberg criteria.

By considering the FOS (n), in both σ_e & σ_m, eqn (2) becomes

$$\frac{\sigma_a}{\sigma_e/n} + \sigma_m/\sigma_{yt}/n = 1 \Rightarrow n \cdot \frac{\sigma_a}{\sigma_e} + n \cdot \frac{\sigma_m}{\sigma_{yt}} = 1$$

$$\boxed{\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{yt}} = \frac{1}{n}} \rightarrow (3)$$ [Soderberg criteria by considering FOS]

By including the fatigue stress concentration factor,

$$K_f \cdot \frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{yt}} = \frac{1}{n}$$ [Note: Strength reduction factor is only for σ_a]

GOODMAN EQUATION:

It is the line drawn between endurance limit and ultimate strength. Here straight line eqn is used

$y = mx + c$; $y = \sigma_a$; $m = a$ } Assumption.
$x = \sigma_m$; $c = b$.

$\therefore$ $\sigma_a = a \cdot \sigma_m + b \rightarrow$ (1)

The conditions are

when $\sigma_a = 0$; $\sigma_m = \sigma_u \rightarrow$ (2a)

$\sigma_m = 0$; $\sigma_a = \sigma_e \rightarrow$ (2b)

Now substituting these conditions in (1).

(2a) in (1) $\Rightarrow$ $0 = a \cdot \sigma_u + b \Rightarrow b = -a \cdot \sigma_u$

(2b) in (1) $\Rightarrow$ $\sigma_e = a(0) + b \Rightarrow b = \sigma_e$

$$\therefore \boxed{a = -\frac{\sigma_e}{\sigma_u}}$$

Now substituting 'a' & b in eqn (1)

$$\sigma_a = \frac{-\sigma_e}{\sigma_u} \cdot \sigma_m + \sigma_e \Rightarrow \sigma_e\left[1 - \frac{\sigma_m}{\sigma_u}\right]$$

$$\frac{\sigma_a}{\sigma_e} = 1 - \frac{\sigma_m}{\sigma_u} \Rightarrow \boxed{\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = 1} \rightarrow (2)$$

Eqn (2) is Goodman eqn.

By considering FOS (n)

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \rightarrow (3)$$

By including ~~fatigue~~ fatigue stress concentration factor.

$$\frac{\sigma_a}{\sigma_e} + K_f \cdot \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \rightarrow (4)$$

MODIFIED GOODMAN RELATION: [GOODMAN YIELD CRITERIA]

Here, yield line is added for designing the life of the components. Any combination of σ_m & σ_a is acting below the region of σ_e, A, σ_{yt}. It gives infinite life.

Following conditions are considered for deriving the Goodman yield criteria.

when $\sigma_m = 0$; $\sigma_a = \sigma_{yp}$ ($\because$ within the yield limit)

when $\sigma_a = 0$; $\sigma_m = \sigma_{yt}$

Now sub these conditions in eqn $\sigma_a = a \cdot \sigma_m + b$

$$\sigma_{yp} = a(0) + b \Rightarrow \boxed{b = \sigma_{yp}}$$

$$0 = a\sigma_{yt} + b \Rightarrow b = -a\sigma_{yt}$$

$$\boxed{a = -\frac{\sigma_{yp}}{\sigma_{yt}}}$$

Now

$$-\frac{\sigma_{yp}}{\sigma_{yt}} = \sigma_{yp} \cdot \sigma_m +$$

$$\sigma_a = -\frac{\sigma_{yp}}{\sigma_{yt}} \cdot \sigma_m + \sigma_{yp}.$$

$$\sigma_a = -\sigma_m + \sigma_y$$ [σ_{yp} & σ_{yt} are just σ_y]

$$\boxed{\sigma_a = \sigma_y - \sigma_m} \rightarrow$$ Modified Goodman relation.

Considering FOS (n)

$$\boxed{\sigma_a = \frac{\sigma_y}{n} - \sigma_m}$$ [FOS is considered only for σ_u, σ_e or σ_y]

GERBER EQUATION:

Gerber uses parabolic curve for finding the life of the component for designing it under fatigue loads. It is a curve drawn between σ_e & σ_u.

parabolic curve for Gerber is

$$\sigma_a = \sigma_e - a\,\sigma_m^2 \rightarrow (1) \qquad [a - \text{constant}]$$

To find 'a', put $\sigma_m = 0$, $\therefore$ [$\cancel{\sigma_m} = \sigma_u$] (Gerber eqn)

$$0 = \sigma_e - a\,\sigma_m^2 \qquad [\sigma_m = \sigma_u]$$

$$0 = \sigma_e - a \cdot \sigma_u^2$$

$$\boxed{a = \frac{\sigma_e}{\sigma_u^2}}$$

put 'a' value in (1)

$$\sigma_a = \sigma_e - \frac{\sigma_e}{\sigma_u^2}\sigma_m^2$$

$$\sigma_a = \sigma_e\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2\right]$$

$$\boxed{\frac{\sigma_a}{\sigma_e} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1} \rightarrow \text{This is the Gerber equation.}$$

By considering FOS,

$$\frac{\sigma_a}{\sigma_e/n} + \frac{\sigma_m^2}{\sigma_u^2/n^2} = 1$$

$$n \cdot \frac{\sigma_a}{\sigma_e} + n^2 \cdot \frac{\sigma_m^2}{\sigma_u^2} = 1$$

By considering the fatigue strength concentration factor

$$\boxed{n \cdot K_f \frac{\sigma_a}{\sigma_e} + n^2 \cdot \frac{\sigma_m^2}{\sigma_u^2} = 1}$$

DESIGN FOR FINITE LIFE: [for simple cases]

$$\sigma_f = 10^c (N)^b$$

(or $0.9\,\sigma_u$)

$$C = \log\left[\frac{(0.8\,\sigma_u)^2}{\sigma_e}\right]$$

$$b = -\frac{1}{3}\log\left[\frac{0.8\,\sigma_u}{\sigma_e}\right]$$

N - no. of cycles, σ_f - fatigue strength.

For calculating the fatigue strength for a particular no. of cycles this formula is used. Here, we use the approximate σ_e value.

1. Calculate the fatigue strength of the steel shaft for a life of 2×10^5 cycles. The ultimate strength & endurance (approximate limit) of the shaft are 300 Mpa and 100 Mpa.

Given:

$N = 2\times10^5$; $\sigma_e = 100$ Mpa.

$\sigma_u = 300$ Mpa

Solution

Fatigue strength is $\sigma_f = 10^c N^b$

$$C = \log\left[\frac{(0.8\times300\times10^6)^2}{100\times10^6}\right] = 8.76$$

$$b = \frac{-1}{3}\log\left[\frac{0.8\times300\times10^6}{100\times10^6}\right]$$

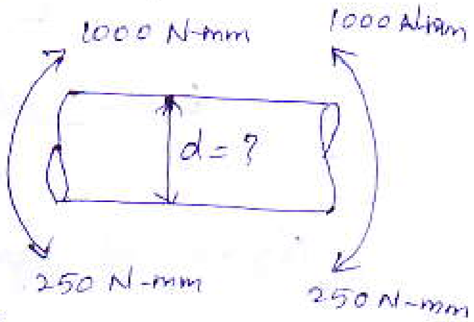
$$b = -0.1267$$

$$\therefore \quad \sigma_f = 10^{8.76}\times(2\times10^5)^{-0.1267}$$

Fatigue strength $(\sigma_f) = 122.56$ Mpa

2. A rod subjected to torsional moment of 1000 N-mm & 250 N-mm. The endurance limit for torsional load is 500 N/mm² & τ_y = 700 N/mm², τ_u = 1200 N/mm². Design the rod using Soderberg, Goodman & Gerber eqn.

[∵ it is torsional problem, we use shear stress].



Given:

$$\tau_e = 500\ N/mm^2$$

$$\tau_y = 700\ N/mm^2 \quad ; \quad \tau_u = 1200\ N/mm^2$$

$$T_{max} = 1000\ N\text{-}mm \quad ; \quad T_{min} = 250\ N\text{-}mm.$$

Solution:

Max. shear stress is

$$\tau_{max} = \frac{T_{max} \cdot r}{J}$$

$$= \frac{1000 \times d/2}{\frac{\pi}{32} d^4}$$

$$\tau_{max} = \frac{5095.54}{d^3}$$

Min. shear stress $\tau_{min} = \frac{T_{min} \cdot r}{J}$

$$= \frac{250 \times d/2}{\pi/32 \cdot d^4}$$

$$\tau_{min} = \frac{1273.89}{d^3}$$

Now

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{5095.54 + 1273.89}{2d^3}$$

$$\tau_m = \frac{3184.715}{d^3}\ N/mm^2$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{5095.54 - 1273.89}{2d^3}$$

$$\tau_a = \frac{1910.825}{d^3} \ N/mm^2$$

(i) Soderberg eqn:

$$\frac{\tau_a}{\tau_e} + \frac{\tau_m}{\tau_y} = \frac{1}{n}$$

[n is not given so take it as 1]

$$\frac{1910.825/d^3}{500} + \frac{3184.715/d^3}{700} = 1$$

$$\boxed{d = 2.03 \ mm}$$

(ii) Goodman eqn:

$$\frac{\tau_a}{\tau_e} + \frac{\tau_m}{\tau_u} = \frac{1}{n}$$

$$\frac{1910.825/d^3}{500} + \frac{3184.715/d^3}{1200} = 1$$

$$\boxed{d = 1.87 \ mm}$$

(iii) Gerber eqn:

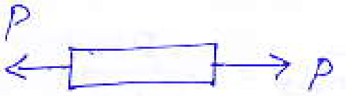
$$\frac{\tau_a}{\tau_e} + n^2 \cdot \frac{\tau_m^2}{\tau_u^2} = 1$$

$$\frac{1910.825/d^3}{500} + 1^2 \cdot \left(\frac{3184.715/d^3}{1200}\right)^2 = 1$$

$$\boxed{d = 1.73 \ mm}$$

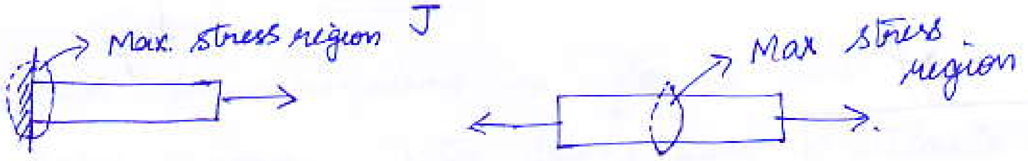
Basics for fatigue & fracture

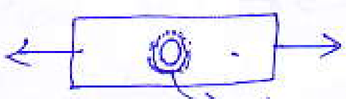
1. Axial Stress $(\sigma) = P/A$

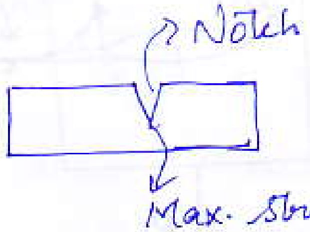


2. Bending Stress $(\sigma) = \frac{M \cdot x}{I}$

3. Shear Stress $(\tau) = \frac{T \cdot r}{J}$







When the cross section of member changes abruptly due to the presence of hole, notch, groove, the stress in those sections of the member does not obey the basic elementary stress equation obtained from the strength of materials (SOM).

Due to the variation in the cross section, the stress will be concentrated in that region, the magnitude of the stress is maximum in that section. This is called stress concentration.

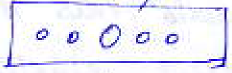
In ductile material, the stress concentration produce elastic deformation and variation in the internal stress flow. In brittle material, it produce sudden fracture.

To reduce stress concentration, we can go for (b) type

(a)

Stress con'n will

(b)

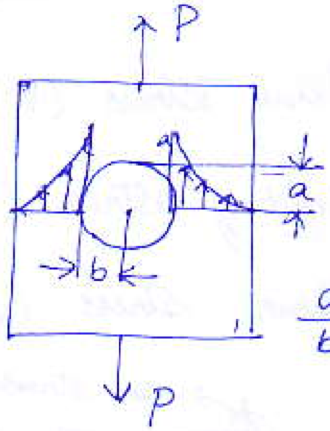


Stress Conc'n will be lower

Stress concentration for circular - elliptical holes:

(i)

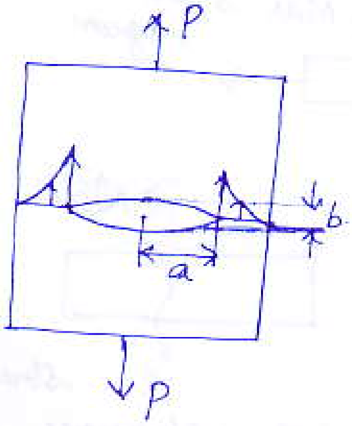
$$\sigma_{max} = \frac{P}{\text{minimum C.S area.}}$$



$\sigma_{max} = 3\sigma_{normal}$.

$\frac{a}{b} = 1$

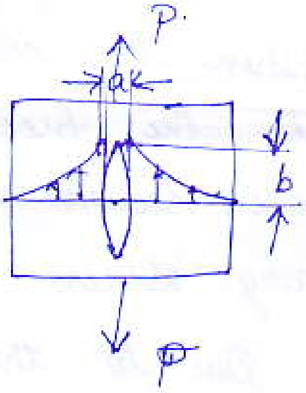
(ii)



$\frac{a}{b} = 2$ $\qquad \sigma_{max} = 5\sigma$

(iii)

$\sigma_{max} = 2\sigma$ $\qquad \frac{a}{b} = \frac{1}{2}$



Reason for Stress concentration:

→ Discontinuity in the section
→ presence of hole, groove & defects
→ Sharp corners.
→ Surface Scratches.

[Defects → internal crack, blow, air holes].

Stress conc'n depends on the size of the object. It also depends on material grain structure. Coarse grain structure has less stress concentration. Fine grain structure has more stress concentration.

→ Brittle materials has more stress concentration than ductile materials. High elastic property material has less stress concentration compared to plastic material.

Stress concentration factor:

$$K_t \cdot \sigma = \sigma_{max}$$

$K_t \rightarrow$ Stress concentration factor.

K_t gives relation b/w maximum stress & normal stress at the area of minimum cross section.

$$K_t = \frac{\sigma_{max}}{\sigma}$$

Stress concentration is found out from photoelastic method experimentally. Data books are available for theoretical values. By using the chart we can calculate the K_t value for different geometries & different loading conditions.

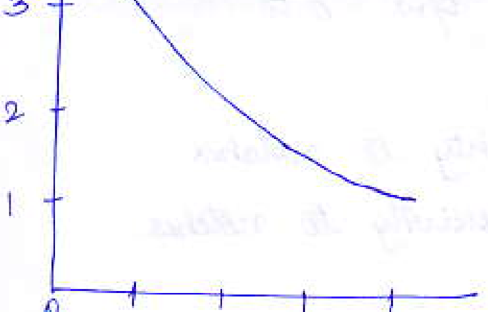
∴ Max. stress for axial, bending & shear stresses are given below

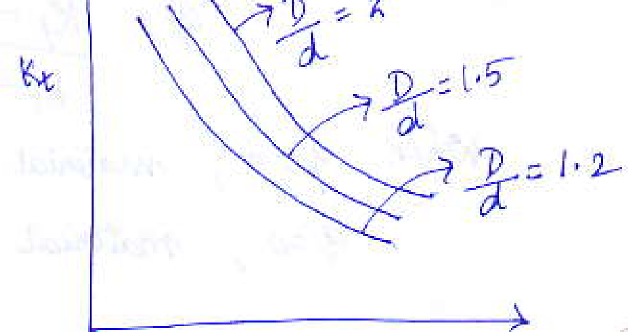
Axial stress $\sigma_{max} = K_t \cdot \sigma_a = K_t \cdot \frac{F}{A}$ where A, I, J are calculated for minimum cross-sectional area.

Bending stress $\sigma_{max} = K_t \cdot \frac{M \cdot x}{I}$

Shear stress $\sigma_{max} = K_t \cdot \frac{T \cdot r}{J}$

Standard chart for stress concentration factor:





Stress concentration for static and dynamic (fatigue) loading:

For static loading:

* Brittle material get sudden fracture compared to ductile material. Hence for brittle material, instead of stress concentration factor, stress intensity factor is used.
* In Ductile material, there is a yielding at the point of excessive stress. Therefore stress conc'n factor will be used.

For dynamic loading:

* stress conc'n factor is a serious problem for both ductile & brittle material. In fatigue loading, K_t is not used, instead of fatigue stress concentration factor K_f is used.

$$K_f < K_t$$

K_F depends upon geometry, material & loading condition whereas K_t only depends upon geometry & loading condition.

$$K_F = \frac{\text{Endurance limit of unnotched Specimen}}{\text{Endurance limit of notched specimen.}}$$

for calculating K_F Value, we are using fatigue testing.

Notch sensitivity index: (q)

It is def'd as the degree to which the theoretical effect of stress conc'n is actually reach the experimental value.

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

$$q = \frac{K_f - 1}{K_t - 1} \quad ; \quad q \text{ lies b/w } 0 \text{ to } 1$$

When $q = 0$, material has no sensitivity to notches

$q = 0$; material has fully sensitivity to notches.

$$K_f = 1 + q(K_t - 1)$$

Notch sensitivity index depends upon geometry, notch radius, size of the section, material, metallurgical factors (heat treatment, grain size, cold working etc) and type of load acting on the material.

Fatigue loads:

It is a cyclic loading acting on the component. It depends upon time. over 90% of failure occurs due to fatigue loads in real components.

Fatigue load acts on leaf springs, crank shaft, a/c structures.

Fatigue failures occurs without any warning. It occurs at a value below the static load for the many no. of cycles.

Types of fatigue loads:

(i) Completely reverse:

$$\sigma_{max} = \sigma_{min}$$

$$\sigma_{mean} = 0$$

(ii) Repeated fatigue load

(iii) Fluctuating load

(iv) Alternating load

(IV) Irregular (or) Random load.

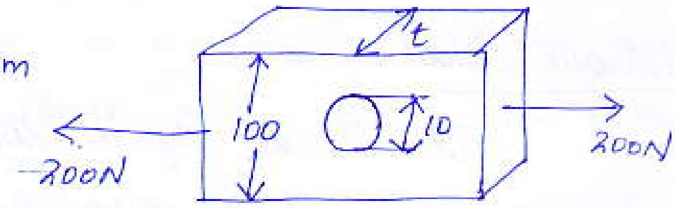
problems:

1. A plate is subjected to tensile force of 200N. The width of the plate is 100mm. It has a central hole of dia 10mm. Determine the thickness of the plate, the tensile stress not exceed the limit of 50 N/mm². Take $K_t = 2.7$

Given:

$\sigma_{max} = 50\ N/mm^2$; $W = 100\,mm$

$K_t = 2.7$ $a = 10$



W.K.T

$$\sigma_{max} = K_t \cdot \sigma_n$$

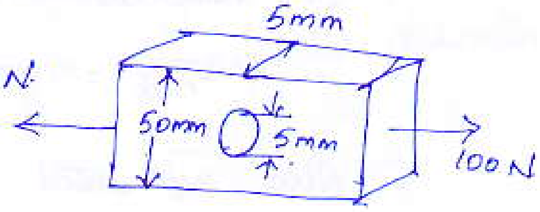
$$\sigma_{max} = K_t \cdot \frac{\text{Load}}{\text{Min c/sal area}}$$

$$50 = 2.7 \times \frac{200}{(W-a)t} \Rightarrow 50 = \frac{2.7 \times 200}{90\,t}$$

$$t = \frac{2.7 \times 2\cancel{0}\cancel{0}}{5\cancel{0} \times 9\cancel{0}} = 0.12\,mm.$$

$$\boxed{t = 0.12\,mm}$$

2. Find the max stress for a plate with central hole of 5 mm; width of the plate 50 mm. Thickness 5 mm. It is subjected to tensile force of 100 N. $K_t = 2.5$.

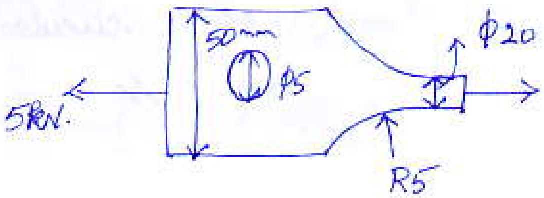


$$\sigma_{max} = K_t \cdot \sigma_n$$

$$= 2.5 \times \frac{100}{(50-5) \times 5}$$

$$\sigma_{max} = 1.1\ N/mm^2.$$

3. A stepped rod subjected to tensile force of 5KN. The ultimate strength of material is 500 N/mm². FOS = 3. Find the max. stress for the rod. Give comments about design. Thickness is 5mm.



Given:

$t = 5mm$

$K_t = 2.5$ for a/D (hole)

$K_t = 1.7$ for d/D (fillet).

Here we have two concentrations.

The Max. stress for hole is

$$\sigma_{max} = K_t \cdot \sigma_n$$

$$= 2.5 \times \frac{5 \times 10^3}{(50-5) \times 5}$$

$$= 55.56\ N/mm^2$$

The max stress for fillet is

$$\sigma_{max} = K_t \cdot \sigma_n$$

$$= \frac{1.7 \times 5 \times 10^3}{20 \times 5} = 85\ N/mm^2$$

The maximum stress value occurs in the fillet region. This should be less than or equal to the permissible stress

$$\text{Permissible stress} = \frac{\text{Ultimate stress}}{\text{Fos}}$$

$$= \frac{500}{3} = 166.67\ N/mm^2$$

$\sigma_{max} \le$ permissible stress. (So, we improve the σ_{max} upto 166.67

Stress concentration for elliptical hole:

The Stress concentration for elliptical hole is derived from the circular hole.

For elliptical hole

$$\sigma_{max} = \sigma\left[1+\frac{2a}{b}\right]$$

Also represented by

$$\sigma_{max} = \sigma\left[1+2\sqrt{a/\rho}\right] \quad \text{where} \quad \rho\,(\text{radius}) = \frac{b^2}{a}$$

$$\sigma_{max} = \sigma\left[1+\frac{2a}{b}\right]$$

For circular holes $a=b$

So $\sigma_{max} = \sigma[1+2]$

$$\sigma_{max} = 3\sigma$$

if $b=0$ [There is a crack like line]

$\sigma_{max} = \infty$, So the Stress concentration factor formula is not suitable for objects having crack.

Formula: (by considering FOS (n) & stress concentration factor (K_f))

Sodberg relation is $K_f\dfrac{\sigma_a}{\sigma_e} + \dfrac{\sigma_m}{\sigma_{yt}} = \dfrac{1}{n}$

Goodman " $\dfrac{\sigma_a}{\sigma_e} + K_f\dfrac{\sigma_m}{\sigma_u} = \dfrac{1}{n}$

Modified Goodman " $\sigma_a = \dfrac{\sigma_y}{n} - \sigma_m$

Gerber " $nK_f\dfrac{\sigma_a}{\sigma_e} + n^2\dfrac{\sigma_m^2}{\sigma_u^2} = 1$

[σ_a - amplitude stress

σ_{yt} = yield strength (Stress)

n - Factor of Safety

σ_m - Mean stress

σ_u = Ultimate Strength

σ_e - Endurance limit.

1. A steel rod subjected to fluctuating axial load that varies from 100 KN (tensile) to 50 KN (Comp). The mechanical properties are $\sigma_u = 1000\ N/mm^2$, $\sigma_y = 600\ N/mm^2$, $\sigma_e = 400\ N/mm^2$. Find the rod dia to give infinite fatigue life. By considering safety factor 2. Use Goodman, Soderberg & Gerber criteria for finding the diameter.

Given

$n = 2$; $\sigma_u = 1000\ N/mm^2$

$\sigma_y = 600\ N/mm^2$; $\sigma_e = 400\ N/mm^2$

$\leftrightarrow \boxed{d = ?} \leftrightarrow$

100 KN - Max load

50 KN - Min load.

Soln:

Max. stress is $\sigma_{max} = \dfrac{\text{Max load}}{\text{area}}$

$$= \frac{100 \times 10^3}{A} = \frac{10^5}{A}\ N/mm^2$$

Min. stress is $\sigma_{min} = \dfrac{\text{Min load}}{\text{Area}} = \dfrac{50 \times 10^3}{A}$

$$\sigma_{min} = \frac{5 \times 10^4}{A}\ N/mm^2$$

(i) Soderberg eqn : ie Design based on Soderberg eqn.

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{yt}} = \frac{1}{n}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{10^5 + 50 \times 10^3}{2A} = \frac{75 \times 10^3}{A}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{10^5 - 5 \times 10^4}{2A} = \frac{25 \times 10^3}{A}$$

$$\frac{25 \times 10^3 / A}{400} + \frac{75 \times 10^3 / A}{600} = \frac{1}{2}$$

$$A = 375 = \frac{\pi}{4} d^2$$

$$\boxed{d = 21.86\ mm}$$

(ii) Goodman eqn

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

$$\frac{25\times10^3/A}{400} + \frac{75\times10^3/A}{1000} = \frac{1}{2} \Rightarrow A = 275 = \frac{\pi}{4}d^2$$

$$\boxed{d = 18.71\,mm}$$

(iii) Gerber eqn:

$$n\cdot\frac{\sigma_a}{\sigma_e} + n^2\cdot\frac{\sigma_m^2}{\sigma_u^2} = 1$$

$$2\times\frac{25\times10^3/A}{400} + 2^2\left(\frac{75\times10^3/A}{1000^2}\right) = 1$$

$$A = 116.35 = \frac{\pi}{4}d^2$$

$$\boxed{d = 12.17\,mm}$$

For giving highest life, Soderberg eqn is used. ($d = 21.86\,mm$)

2. Find the thickness of the plate which is having 50 mm width & 100mm length is Subjected to continous load of 300 kN & 50 kN, Take $\sigma_e = 200\,N/mm^2$, $\sigma_y = 300\,N/mm^2$, $\sigma_u = 500\,N/mm^2$, $n = 2.5$. Fatigue strength detuction factor $K_f = 1$.

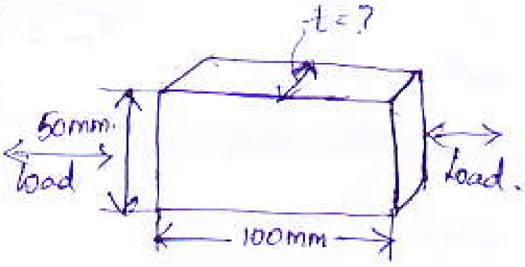
Given:

$W = 50\,mm$; $\sigma_e = 200\,N/mm^2$,

$l = 100\,mm$; $\sigma_y = 300\,N/mm^2$,

$t = ?$, $\sigma_u = 500\,N/mm^2$,

$n = 2.5$



Solution:

Maximum stress is $\sigma_{max} = \frac{\text{Max load}}{\text{Area}}$ $[A = W\times t]$

$$\sigma_{max} = \frac{300\times10^3}{50\times t}$$

$$\sigma_{max} = 6000/t\ N/mm^2$$

Minimum stress is $\sigma_{min} = \dfrac{\text{Min. load}}{\text{Area}}$

$$= \frac{50 \times 10^3}{50 \times t}$$

$$\sigma_{min} = \frac{1000}{t} \ N/mm^2$$

Mean stress, $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

$$\sigma_m = \frac{6000 + 1000}{2t}$$

$$\sigma_m = \frac{3500}{t}$$

Amplitude stress (σ_a)

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$= \frac{6000 - 1000}{2t} = \frac{5000}{2t}$$

$$= \frac{2500}{t}$$

(i) Soderberg equation:

$$K_f \cdot \frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{yt}} = \frac{1}{n}$$

$$1 \cdot \frac{250\cancel{0}}{20\cancel{0}t} + \frac{35\cancel{00}}{3\cancel{00}t} = \frac{1}{2.5}$$

$$\boxed{t = 60.42 mm}$$

(ii) Goodman equation:

$$\frac{\sigma_a}{\sigma_e} + K_f \cdot \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

$$\frac{25\cancel{00}}{2\cancel{00}t} + 1 \cdot \frac{\overset{7}{\cancel{3500}}}{\cancel{500}t} = \frac{1}{2.5}$$

(iii) Gerber equation:

$$n \cdot K_f \cdot \frac{\sigma_a}{\sigma_e} + n^2 \cdot \frac{\sigma_m^2}{\sigma_u^2} = 1$$

$$2.5(1) \cdot \frac{2500}{200t} + 2.5^2 \cdot \left(\frac{3500}{500t}\right)^2 = 1$$

$$2.5\left(\frac{25}{2t}\right) + 2.5^2 \times \frac{7^2}{t^2} = 1$$

$$\boxed{t = 39.08\,mm}$$

So Soderberg's design gives more life for the component.

3. A component subjected to varying bending stress of 150 N/mm² and 45 N/mm². Find the ultimate strength required for the material by using Soderberg, Gerber and Goodman formula.

Assume $\sigma_y = 0.5\sigma_U$; $\sigma_e = 0.3\sigma_U$; $n = 1$

Given:

$\sigma_{max} = 150\,N/mm^2$; $\sigma_y = 0.5\sigma_U$; $\sigma_y < \sigma_e$

$\sigma_{min} = 45\,N/mm^2$; $\sigma_e = 0.3\sigma_U$

Solution:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{150+45}{2} = \frac{195}{2} = 97.5\,N/mm^2$$

$$\boxed{\sigma_m = 97.5\,N/mm^2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{150-45}{2} = \frac{105}{2}$$

$$\boxed{\sigma_a = 52.5\,N/mm^2}$$

(i) Sodarberg eqn:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{yt}} = \frac{1}{n}$$

$$\frac{52.5}{0.3\,\sigma_u} + \frac{97.5}{0.5\,\sigma_u} = \frac{1}{1}$$

$$\sigma_u = \frac{52.5}{0.3} + \frac{97.5}{0.5}$$

$$= 175 + 195$$

$$\boxed{\sigma_u = 370\ N/mm^2}$$

(ii) Goodman eqn:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

$$\frac{52.5}{0.3\,\sigma_u} + \frac{97.5}{\sigma_u} = 1 \Rightarrow 175 + 97.5 = \sigma_u$$

$$\boxed{\sigma_u = 272.5\ N/mm^2}$$

(iii) Gerber eqn:

$$n.\frac{\sigma_a}{\sigma_e} + n^2.\frac{\sigma_m^2}{\sigma_u^2} = 1$$

$$1\left(\frac{52.5}{0.3\sigma_u}\right) + 1^2\left(\frac{97.5^2}{\sigma_u^2}\right) = 1$$

$$\frac{175}{\sigma_u} + \frac{9506.25}{\sigma_u^2} = 1 \Rightarrow \sigma_u^2 = 175\sigma_u + 9506.25$$

$$\frac{175\,\sigma_u^2 + 9506.25\,\sigma_u}{\sigma_u^3} = 1$$

$$175\,\sigma_u^2 + 9506.25\,\sigma_u = \sigma_u^3$$

$$\boxed{\sigma_u = 218.5\ N/mm^2}$$

(iv) Modified Goodman Eqn:

$$\sigma_a = \frac{\sigma_y}{n} - \sigma_m$$

$$52.5 = \frac{0.5\sigma_u}{1} - 97.5 \Rightarrow 52.5 + 97.5 = 0.5\sigma_u$$

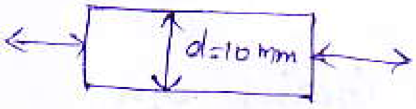
$$\sigma_u = \frac{150}{0.5} = 300\ N/mm^2$$

$$\boxed{\sigma_u = 300\ N/mm^2}$$

∴ From the above relations, Soderberg eqn has higher ultimate strength. So Soderberg criteria is the safest design.

4. Find the Max. load, min. load and amplitude stress for a rod subjected to fatigue load of $\sigma_m = 500\ N/mm^2$, using Soderberg, Goodman, Modified Goodman & Gerber eqn. Take $\sigma_u = 1000\ N/mm^2$; $\sigma_y = 600\ N/mm^2$; $\sigma_e = 300\ N/mm^2$. State design criteria; $n = 1$

Given:



$\sigma_m = 500\ N/mm^2$

$\sigma_u = 1000\ N/mm^2$

$\sigma_y = 600\ N/mm^2$

$\sigma_e = 300\ N/mm^2$; $n = 1$

$$A = \frac{\pi}{4}(10)^2$$

$$A = 78.5\ mm^2$$

Solution:

(i) Soderberg eqn:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \Rightarrow \frac{\sigma_a}{300} + \frac{500}{600} = \frac{1}{1}$$

$$\frac{\sigma_a}{300} + \frac{500}{600} = 1 \Rightarrow 2\sigma_a + 500 = 600$$

$$2\sigma_a = 100 \Rightarrow \boxed{\sigma_a = 50\ N/mm^2}$$

(ii) Goodman eqn:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \Rightarrow \frac{\sigma_a}{300} + \frac{500}{1000} = 1$$

$$\frac{\sigma_a}{300} = 1 - \frac{500}{1000} = \frac{500}{1000} = 0.5$$

$$\sigma_a = 0.5(300) = 150\ N/mm^2 \quad \boxed{\sigma_a = 150\ N/mm^2}$$

(iii) Modified Goodman:

$$\sigma_a = \frac{\sigma_y}{n} - \sigma_m = \frac{600}{1} - 500 = 100\ N/mm^2$$

$$\boxed{\sigma_a = 100\ N/mm^2}$$

(iv) Gerber eqn:

$$n.\frac{\sigma_a}{\sigma_e} + n^2.\frac{\sigma_m^2}{\sigma_u^2} = 1$$

$$1.\frac{\sigma_a}{300} + 1^2.\frac{500^2}{1000^2} = 1$$

$$\frac{\sigma_a}{300} = 1 - \frac{500^2}{1000^2} = 1 - \left(\frac{500}{1000}\right)^2 = 1 - (0.5)^2$$

$$\frac{\sigma_a}{300} = 0.75 \Rightarrow \boxed{\sigma_a = 225\ N/mm^2}$$

So

$$\left.\begin{aligned} \sigma_{max} &= \sigma_m + \sigma_a = 500 + 50 = 550\ N/mm^2 \\ \sigma_{min} &= \sigma_m - \sigma_a = 500 - 50 = 450\ N/mm^2 \end{aligned}\right\} \text{Soderberg eq}$$

$$\left.\begin{aligned} \sigma_{max} &= 500 + 150 = 650\ N/mm^2 \\ \sigma_{min} &= 500 - 150 = 350\ N/mm^2 \end{aligned}\right\} \text{Goodman eqn}$$

$$\left.\begin{aligned} \sigma_{max} &= 500 + 100 = 600\ N/mm^2 \\ \sigma_{min} &= 500 - 100 = 400\ N/mm^2 \end{aligned}\right\} \text{Modified Goodman eqn}$$

$$\left.\begin{aligned} \sigma_{max} &= 500 + 225 = 725\ N/mm^2 \\ \sigma_{min} &= 500 - 225 = 275\ N/mm^2 \end{aligned}\right\} \text{Gerber eqn.}$$

$\therefore$ Maximum load is $P_{max} = \dfrac{\sigma_{max}}{C/s\ area}$

Minimum load is $P_{min} = \sigma_{min}$

Soderberg eqn:

$$P_{max} = \frac{550}{78.5} = 7N$$

$$P_{min} = \frac{450}{78.5} = 5.73N$$

Goodmann eqn:

$$P_{max} = \frac{650}{78.5} = 8.3N$$

$$P_{min} = \frac{350}{78.5} = 4.46N$$

Modified Goodmann eqn:

$$P_{max} = \frac{600}{78.5} = 7.64N$$

$$P_{min} = \frac{400}{78.5} = 5.10N$$

Gerber eqn:

$$P_{max} = \frac{725}{78.5} = 9.235N$$

$$P_{min} = \frac{275}{78.5} = 2.87N.$$